Hybrid Simulation: Similitude

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Dimensional Analysis

 Qualitative understanding of physical processes

 Basic dimensions (of interest for dynamics of structures):
  - Length (L)
  - Force (F)
  - Time (T)

 Results in non-dimensional factors (Pi-factors) that relate basic dimensions to express the physics of the problem
Scale Factors

- Scale factor
  - Length-force relation:
    - Dictated by preserving the scale of modulus of elasticity (stress)
  - Gravity-time relation:
    - Gravity is the same
    - Effect of gravity may be important
  - Sometimes, we can neglect effects of gravity loads on response
    - Preserve mass density

\[ S = \frac{D_{\text{prototype}}}{D_{\text{model}}} \]

\[ S_F = S_L^2; \quad S_E = 1 \]

\[ S_g = 1; \quad S_T = \sqrt{S_L} \]

\[ S_\rho = 1; \quad S_T = S_L \]
Summary: Earthquake Testing

Table 2.11 Summary of Scale Factors for Earthquake Response of Structures

<table>
<thead>
<tr>
<th>(1)</th>
<th>Dimension (3)</th>
<th>Scale Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2)</td>
<td></td>
<td>True Replica Model (4)</td>
</tr>
<tr>
<td>Loading</td>
<td></td>
<td>$F$</td>
</tr>
<tr>
<td>Force, $Q$</td>
<td></td>
<td>$F L^{-2}$</td>
</tr>
<tr>
<td>Pressure, $q$</td>
<td></td>
<td>$F L^{-2}$</td>
</tr>
<tr>
<td>Acceleration, $a$</td>
<td>$L T^{-2}$</td>
<td>1</td>
</tr>
<tr>
<td>Gravitational acceleration, $g$</td>
<td>$L T^{-2}$</td>
<td>1</td>
</tr>
<tr>
<td>Velocity, $v$</td>
<td>$L T^{-1}$</td>
<td>$S_f^{1/2}$</td>
</tr>
<tr>
<td>Time, $t$</td>
<td>$T$</td>
<td>$S_f^{1/2}$</td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
<td>$S_f^i$</td>
</tr>
<tr>
<td>Linear dimension, $l$</td>
<td>$L$</td>
<td>$S_l$</td>
</tr>
<tr>
<td>Displacement, $\delta$</td>
<td>$L$</td>
<td>$S_l$</td>
</tr>
<tr>
<td>Frequency, $\omega$</td>
<td>$T^{-1}$</td>
<td>$S_f^{-1/2}$</td>
</tr>
<tr>
<td>Material properties</td>
<td>$FL^{-2}$</td>
<td>$S_E$</td>
</tr>
<tr>
<td>Modulus, $E$</td>
<td>$FL^{-2}$</td>
<td>$S_E$</td>
</tr>
<tr>
<td>Stress, $\sigma$</td>
<td></td>
<td>$S_E$</td>
</tr>
<tr>
<td>Strain, $\varepsilon$</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>Mass density, $\rho$</td>
<td>$FL^{+1} T^2$</td>
<td>$S_E / S_l$</td>
</tr>
<tr>
<td>Energy, $E_N$</td>
<td>$FL$</td>
<td>$S_E S_f^3$</td>
</tr>
</tbody>
</table>

* $(gpl/E)_m = (gpl/E)_p^*.$

**Harris and Sabnis textbook**
Constraints and Goals

**Constraints:**
- Laboratory size
- Actuator force
- Reaction capacity
- Material (E and mass density)

**Goals:**
- Enable economical and realistic hybrid simulation of seismic response of structures
- Usually, smaller is cheaper
Mass density is key

Computer models do not have a problem

Physical models:

- Complete similitude (consistent scaling of gravity acceleration):

\[ S_T = \sqrt{S_L} \]

- Distorted model (still under gravity):

\[ S_T = S_L \]

Analysis by Kumar et.al.
Gravity Matters
(Procedure 1)

Preserve mass density

Scale mass

May conduct test on:

- Prototype
  - Obtain target displacement, scale it down, measure force, scale it up

- Model
  - Scale excitation (amplitude and duration), solve, scale up the response

Methods are equivalent

\[ S_\gamma = 1; \quad S_M = S_L^3 \]
Gravity does not Matter
(Procedure 2)

❖ Choose not to scale time
❖ Force and length factors remain
❖ May conduct tests on:
  ■ Prototype
  ■ Model:
    ❖ If test is done on the model, the amplitude of ground motions must be scaled by length scale

\[ S_T = 1 \]
A Comparison

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Dimensions</th>
<th>Procedure 1</th>
<th>Procedure 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Length</td>
<td>$L$</td>
<td>$(S)$</td>
<td>$(S)$</td>
</tr>
<tr>
<td>Mass</td>
<td>$M$</td>
<td>$(S^3)$</td>
<td>$S$</td>
</tr>
<tr>
<td>Time</td>
<td>$T$</td>
<td>$S$</td>
<td>$(1)$</td>
</tr>
<tr>
<td>Stress</td>
<td>$ML^{-1}T^{-2}$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Velocity</td>
<td>$LT^{-1}$</td>
<td>1</td>
<td>$S$</td>
</tr>
<tr>
<td>Acceleration</td>
<td>$LT^{-2}$</td>
<td>$1/S$</td>
<td>$S$</td>
</tr>
<tr>
<td>Force</td>
<td>$MLT^{-2}$</td>
<td>$(S^2)$</td>
<td>$(S^2)$</td>
</tr>
<tr>
<td>Stiffness</td>
<td>$MT^{-2}$</td>
<td>$S$</td>
<td>$S$</td>
</tr>
<tr>
<td>Damping</td>
<td>$C = 2\xi\sqrt{KM}$</td>
<td>$MT^{-1}$</td>
<td>$S^2$</td>
</tr>
<tr>
<td>Natural frequency $\omega$</td>
<td>$T^{-1}$</td>
<td>$1/S$</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: Scale factor for quantity $i$; $S_i = i_p/i_m$; items in parentheses indicate specified factors.

Kumar et.al.
Note on GM Scaling

Many hybrid simulations conducted using Procedure 2 on the model are done without scaling the ground motion amplitude:

- This is equivalent to using an $S_L$ times stronger ground motion record.
- The structure may fail prematurely.
- The response may be highly non-linear.
Similitude and Substructures

- Scaling to measured response and computed response
- Scale for substructures need not be the same!
Similitude and SubStructures

- **Scaling must be consistent:**
  - It is not necessary that the sub-structures have the same scale
  - It is, however, important that the their state data is correctly scaled \textit{wrt.} the integrator scale

- **Errors scale, too!**
  - instruments have fixed error and sensitivity, thus it pays to use as large-scale physical models as possible
Example: Bridge SSI

Bridge deck: computer model
SL=1
ST=1

Foundation: physical model (centrifuge)
SL=81
ST=9

Column: physical model (structures lab)
SL=1
ST=1
Thank you!

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